"A METHODOLOGY FOR THE DEVELOPMENT OF A COUPLING MODEL FOR HIGH ENERGY LASER INTERACTIONS WITH METALS "

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ABSTRACT

Future Combats Systems are actively considering the use of a high-energy laser (HEL) for various missions. The laser of choice for this mission is likely to be a solid-state laser operating in the one micron wavelength region. Accordingly, lethality studies are underway to assess the utility of these HELs. The potential for dramatic increases in kill efficiency, as a function of various weapons parameters will be an important input to the selection and development of supporting technologies for an HEL weapon. Accordingly, the U.S. Army Space and Missile Defense Command (SMDC) is actively investigating the development of a model for the kill efficiency.

This paper will report on the initial phase of this study and emphasizes the methodology for the development of a coupling model for materials of interest to the Army. Using continuous wave (CW) lasers operating at various powers and wavelengths as a source of data, a methodology is developed for inverting the non-linear partial differential equation solution for laser interaction with metals. Experimental studies of this laser interaction with metals has been performed using the 100kW CO_2 Laser at the Laser Hardened Materials Evaluation Laboratory (LHMEL) and the 20 KW solid state fiber laser at the Air Force Research Laboratory located at Kirtland Air Force Base. These experiments included a variety of tests and extensive diagnostics operating at infrared and visible wavelengths. The methodology for developing a coupling model is derived analytically and comparison with experimental data is illustrated.

1. BACKGROUND

The irradiance from a laser impinging on a plate of material is generally modeled as a heat flux. The irradiance of the laser is modeled in the target plane as a spatial function of x and y; f(x,y) as in figure 1. This irradiance profile is incident on the plate in figure 1. The heat flux is the fraction of incident irradiance that is absorbed by the plate. The fraction is termed the coupling of the laser light to the material. In general, the coupling may be a function of x , y and t (time), and will be denoted as q(x,y,0,t) (in this case, z=0). The objective of the methodology described herein is to determine q. The ultimate objective is to determine q, not as a function of t, but of temperature, irradiance, or other appropriate parameter. This paper does not address this last objective.
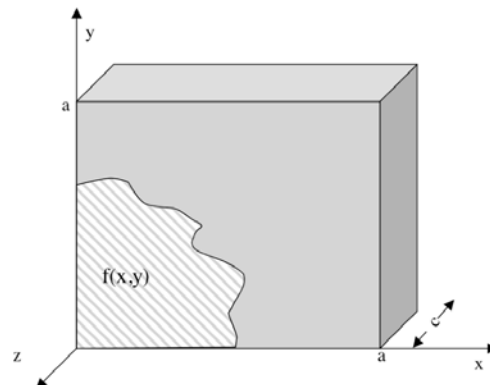

Figure 1. Problem Description

The coordinate system used throughout this paper is indicated in figure 1. The variables are:

T - temperature
z - coordinate in the thickness direction
q - absorbed flux
c - plate thickness
κ - thermal diffusivity
λ - eigenvalue for the solution, with subscript
t - time

Testing is most effective, both from a cost and technical viewpoint, when the data reduction is simple, i.e. when the parameter space is minimized. For example, it is important that the

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irradiance function, f(x,y), be relatively independent of time. Cyclical high frequency fluctuations are usually acceptable[1], but if they are severe (>20% amplitude variations) they need to be addressed on a case-by-case basis. A key assumption is that, once a coupling model is developed, time variations that may occur in field situations are easily accommodated.

Another complexity that occurs is other sources of heat loss or gain that may occur in the experiment. Fortunately, laser power overwhelms heat loss terms that occur due to radiation, convection, and conduction unless the tested irradiance is at the low extreme of the parameter space. These irradiances are usually not of much interest for the obvious reason that, if the HEL effect is competing with other loss terms, then it is not creating much of an effect at the target. There are a few exceptions to this and they must be handled accordingly. An example might be ablative, carbon based, materials that reach a high temperature at the ablative surface causing significant heat loss from radiation that competes somewhat with the lower irradiances.

The complexities relating to temporal variations and heat loss are relatively easily handled with modern CW lasers. These lasers are well characterized temporally and the powers that they operate at afford the opportunity to test at irradiances high enough to overwhelm other sources of heat loss. The problem reduces, then, to one of correcting for the spatial variation of the HEL irradiance. For a variety of practical reasons, the most relevant diagnostic for determining the coupled irradiance function, q(x,y,0,t), is the spatial and temporal temperature function on the back surface of the plate. The problem is further complicated by the fact that a specific temperature on a point on the rear surface of the plate does not uniquely determine the irradiance on the front surface that produced that temperature[2]. The analysis that follows describes a method for overcoming this problem.

2. ANALYSIS

2.1 The One Dimensional Solution

Recalling the assumptions from section one, we assume the relatively simple boundary conditions of an insulated plate, uniformly irradiated on the front surface by a laser as shown in figure 2. It is evident that the problem,
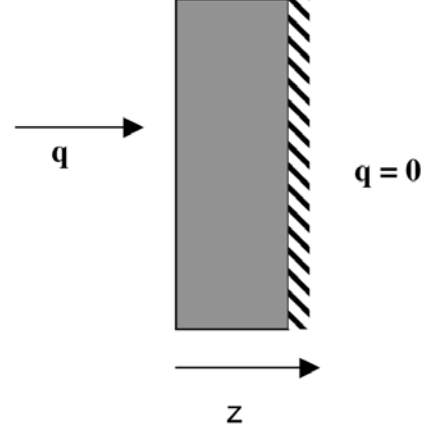

Figure 2. One dimensional problem

temperature solution, as there is no accompanying heat loss associated with the heat gain from the laser. There is, however, a steady state heat flux solution, q(z). Since the boundary conditions are in terms of flux, a modified form of the heat transfer equation[3] will be used:

$$\rho c_p \frac{\partial \vec{q}}{\partial t} = k \frac{\partial^2 \vec{q}}{\partial z^2} \rightarrow k \frac{\partial^2 q_z}{\partial z^2} = 0 \quad (1)$$

The solution to (1) is straightforward:

$$-k \frac{\partial T(z,t)}{\partial z} = q = b_1 z + b_2 \quad \text{or}$$

$$T(z,t) = \frac{b_1}{2} z^2 + b_2 z + b_3 \quad (2)$$

In (2), the variables **b** are constants of integration. The implication of (2) is that when q is at its steady-state value, the temperature distribution is parabolic. Of the three constants, only the last one may be a function of time. The initial condition for T(z,0) is zero. Fitting these conditions to the solution above yields the following, transient solution that meets all of the boundary conditions:

$$T(z,t) = \frac{q_0}{c} \left[\begin{array}{c} \dfrac{z^2}{2} - cz + \dfrac{c^2}{3} + \kappa t - 2 \\ \displaystyle\sum_{p=1}^{\infty} \dfrac{\cos(\lambda_p z)}{\lambda_p^2} \exp(-\lambda_p^2 \kappa t) \end{array} \right] \quad (3)$$

Where:

$$\lambda_p = \frac{p\pi}{c} \qquad (4)$$

2.2 The Three Dimensional Solution

The same approach may be used for the three dimensional solution, that is solving the, now, vector form of the heat conduction equation:

$$\rho c_p \frac{\partial \vec{q}}{\partial t} = k \frac{\partial^2 \vec{q}}{\partial x^2} + k \frac{\partial^2 \vec{q}}{\partial y^2} + k \frac{\partial^2 \vec{q}}{\partial z^2} = k\nabla^2 \vec{q} \qquad (5)$$

As in 1.1, a quasi-steady solution for this equation can be determined:

$$kT(x,y,z,t) = \frac{1}{c}\sum_{n,m=0}\sum \frac{\alpha_{nm}}{\lambda_{nm}\ \sinh(\lambda_{nm}c)}$$
$$\left(\cosh[\lambda_{nm}(c\text{-}z)]\cos(\lambda_n x)\cos(\lambda_m y)\right) \qquad (6)$$
$$+\bar{q}\left(\frac{z^2}{2} - cz + \frac{c^2}{3} + \kappa t\right)$$

Assuming the same initial conditions as in 1.1, the complete solution that satisfies all of the boundary conditions is then:

$$kT(x,y,z,t) = \frac{1}{c}\Bigg[\sum_{n,m=0}\sum \frac{\alpha_{nm}}{\lambda_{nm}\ \sinh(\lambda_{nm}c)}$$
$$\left(\cosh[\lambda_{nm}(c\text{-}z)]\cos(\lambda_n x)\cos(\lambda_m y)\right) +$$
$$\bar{q}\left(\frac{z^2}{2} - cz + \frac{c^2}{3} + \kappa t\right) - \qquad (7)$$
$$\sum_{n,m=1}^{\infty}\sum \frac{\alpha_{nm}}{\lambda_{nm}^2}\ \sinh(\lambda_{nm}c)$$
$$\cos(\lambda_n x)\cos(\lambda_m y)\exp(\text{-}\lambda_{nm}^2\kappa t) -$$
$$2\sum_{n,m=0}^{\infty}\sum\sum_{p=1}\frac{\alpha_{nm}}{\lambda_{nmp}^2}\sinh(\lambda_{nm}c)\ \cos(\lambda_n x)$$
$$\cos(\lambda_m y)\cos(\lambda_p z)\exp(\text{-}\lambda_{nmp}^2\kappa t) -$$
$$2\bar{q}\sum_{p=1}^{\infty}\frac{\cos(\lambda_p z)\exp(-\lambda_p^2\kappa t)}{\lambda_p^2}\Bigg]$$

Where:

$$\lambda_{nmp}^2 = \lambda_m^2 + \lambda_n^2 + \lambda_p^2 \text{ where}$$
$$\lambda_n = \frac{n\pi}{a} \text{ and } \lambda_m = \frac{m\pi}{a} \qquad (8)$$

The solution in 1.1 will now be compared with the above solution.

2.3 Solution Comparison

Equation (7) is clearly quite complex but it can be simplified by analyzing it on a mode-by-mode basis. The terms:

$$\cos(\lambda_m x) \text{ and } \cos(\lambda_n y) \qquad (9)$$

result from the eigenfunctions of q(x,y) and equation (7) can easily be assessed on an eigenfunction basis. The equations are, after all, linear in q(x,y). So, let:

$$q(x,y) = \cos(\lambda_1 x)\cos(\lambda_1 y) \qquad (10)$$

and equation (7) reduces to:

$$kT(x,y,z,t) = q(x,y)\begin{bmatrix}\dfrac{\cosh[\lambda_{11}(c\text{-}z)]}{c\lambda_{11}^2\ \sinh(\lambda_{11}c)} - \dfrac{1}{c\lambda_{11}^2}\exp(\text{-}\lambda_{11}^2\kappa t) - \\ \dfrac{2}{c}\sum_1^{\infty}\dfrac{\cos(\lambda_p z)\exp(\text{-}\lambda_{11p}^2\kappa t)}{\lambda_{11p}^2}\end{bmatrix} \qquad (11)$$

The next step in the comparison of the two solutions is to take the time derivative of both (3) and (11). This step will illustrate some important physics involved in the process. The time derivative of (3) is:

$$k\frac{\partial T(z,t)}{\partial t} = q_0\left[\frac{-\kappa}{c} + \frac{2\kappa}{c}\sum_{p=1}^{\infty}\cos(\lambda_p z)\exp(-\lambda_p^2\kappa t)\right] \qquad (12)$$

while the time derivative of (11) is:

$$k\frac{\partial T(x,y,z,t)}{\partial t} = q(x,y)\exp(\text{-}\lambda_{11}^2\kappa t)$$
$$\left[\frac{-\kappa}{c} + \frac{2\kappa}{c}\sum_1^{\infty}\cos(\lambda_p z)\exp(-\lambda_p^2\kappa t)\right] \qquad (13)$$

The ratio of the two solutions, at any point (x_0, y_0, z_0) in the plate is given by the rather simple expression:

$$\exp(\text{-}\lambda_{11}^2\kappa t) \qquad (14)$$

or, recalling the expression for λ:

$$\exp(-\frac{\pi^2}{2a^2}\kappa t) \qquad (15)$$

The interpretation of (15) is very interesting. By inspection, it shows that, as t goes to zero, the solution becomes one-dimensional. It shows that the heat flow can be modeled as one-dimensional as long as (15) is close to one, meaning as long as "t" is small and "a" is large. Note that this result is completely independent of the thickness, "c", of the plate.

The integral of (15) sums the difference between the one-dimensional and three-dimensional solutions for this flux distribution to get the total temperature difference:

$$\frac{2a^2}{\tau\pi^2\kappa}\left[1-e^{-\frac{\pi^2}{2a^2}\kappa t}\right] \qquad (16)$$

Expression (16) applies to the flux distribution in (10) and it highlights the functional dependency. Application to other flux distributions must be assessed. The more common, and more desirable, flux distribution in test devices is the flattop for which the flux is a constant value within a given spot radius and equals zero outside that radius.

The flux distribution in (10) falls off as a cosine function from the center of the distribution while a flattop distribution does not decrease at all until the abrupt change at the top-hat radius. Therefore, at the center of the spot, the temperature should be more one-dimensional than indicated by (16), while the distribution at the spot edge should be less one-dimensional. It can be determined, empirically, that the temperature distribution at the center of the spot is within 99% of the one-dimensional solution when (16) is greater than .95.

3. APPLICATION

3.1 Methodology

From the analysis in section 2 it is evident that the problem of determining the flux from the temperature distribution is greatly simplified when the problem can be assumed to be one-dimensional. In fact, if the test is conducted so that (16) is greater than .95, the problem is linear (material properties constant with temperature),

and the coupling function is constant in time, then the problem clearly has a unique solution which is easily invertible.

In general, though, the problem is not linear in a relevant parameter space. Lasers are of interest because they create nonlinear effects in materials, i.e. phase changes. In addition, a coupling function that is constant in time may not be assumed. It can be argued that (16) still applies despite the nonlinearity. As a material approaches phase change, the specific heat of the material rises rapidly. As a result, the thermal diffusivity declines rapidly resulting in much lower heat conduction so that (16) is somewhat conservative. The specific heat curve for typical steel is shown in figure 3.
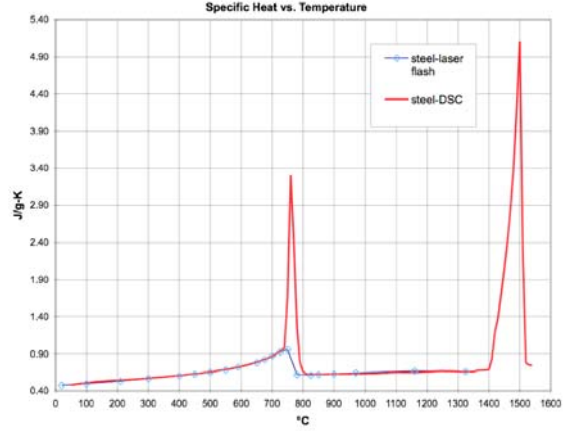

Figure 3. Typical specific heat versus temperature plot for steel

It can also be argued that, to first order, the heat conduction in the x-y plane is independent of thickness. The one-dimensional behavior of the material where the z coordinate is beyond the melt region is, thus, not strongly affected by the nonlinear effects occurring in the melt region. This has been verified by numerical calculations where the 95% criteria for (16) remains valid all of the way to melt-thru. In any case, the non-linearity of the problem does argue that the problem should be done numerically.

The process is, therefore, as follows:

- Verify the experimental data satisfies the one-dimensional in (16)
- Solve a series of non-linear partial differential equations at a sequence of irradiances
- Store the temperature-time sequences in

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a matrix
- Interpolate with the measured temperature sequence (invert the equation) based on an assumed coupling model
- Iterate the coupling model until a solution of the desired accuracy is attained

3.2 Results

A typical assumed coupling modeled is illustrated in figure 4. The rationale behind such a model is that it makes some sense physically and, given the other uncertainties of the problem, has the desired accuracy. Most materials in common use are not optically clean. Surface impurities, coatings, and other surface imperfections dominate the coupling function. This remains true until a certain temperature is attained on the front surface at which the surface affects begin to change. The change continues until the surface reaches some steady value, which is determined by many factors such as irradiance, temperature, or wind flow (to name a few).
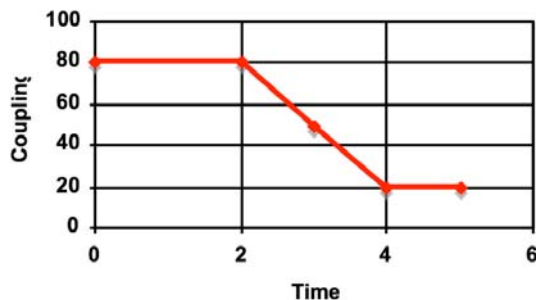
Figure 4. Sample trial coupling function

The procedure for the process in 3.1 has been automated in order to process large amounts of data so that statistical effects can be determined. An example of the resulting data fit is illustrated in figure 5. As can be seen, the fit is quite good. It is now up to the researcher to determine the root physical causes for the coupling function behavior and to construct a physical model for the behavior.
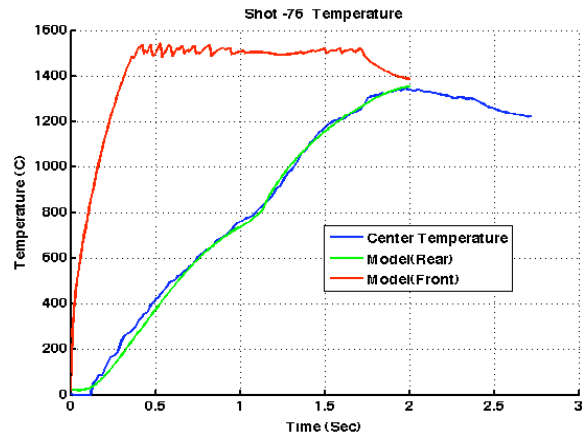
Figure 5. A comparison of the method described with experimental data

CONCLUSIONS

A method, based on the physics of the interaction of laser light with materials, has been illustrated. The analytical assessment of the method provides not only a methodology but a better understanding of the dimensional effects of the interaction. The analytical solution provides a practical method for planning experiments and reducing data quickly resulting in cost effective and productive laser testing.

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[1] "Laser Diagnostics by Heat Conduction", C.R. LaMar, AIAA/ASME 4th Joint Thermophysics and Heat Transfer Conference, 1986
[2] This is a practical, rather than a theoretical, assertion. There is a practical limit to the accuracy and resolution of the temperature measurement on the rear surface. It is relative straightforward to show different irradiance functions that can produce the same temperature on the back surface within the accuracy of the temperature measurement.
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